Higher-order Fourier analysis and related topics

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1 List of Topics

Monday (Foundations)

1. **[Stefanos Lappas] Roth's theorem**
   Text book: https://yufeizhao.com/gtacbook/
   Remarks: Study Chapter 6 up to Section 6.4. Prove Theorem 6.4.1.

2. **[Hanaë Vandanjon] Szemerédi's regularity lemma and the triangle removal**
   Text book: https://yufeizhao.com/gtacbook/
   Remarks: Prove Theorem 2.1.9 and Theorem 2.3.1.

3. **[Deepanshu Kush] New proofs of Plünnecke-type estimates for product sets in groups**
   Giorgis Petridis
   Journal link: https://link.springer.com/article/10.1007/s00493-012-2818-5
   Balog–Szemerédi–Gowers theorem
   Textbook: https://yufeizhao.com/gtacbook/
   Remarks: Present Sections 2 and 3 of the paper, culminating in the proof of Theorem 1.2. Concerning the book, study Section 7.13 and prove Theorem 7.13.6.

4. **[Shota Uka] Freiman's theorem**
   Text book: https://yufeizhao.com/gtacbook/
   Remarks: Prove Theorem 7.5.4 (Freiman's theorem in groups with bounded exponent). To this end, study Sections 7.1-7.5. Skip sum set things already covered in Talk 3.

5. **[António Rocha Neves] On the Bogolyubov—Ruzsa lemma**
   Tom Sanders
   Journal link: http://dx.doi.org/10.2140/apde.2012.5.627
   Remarks: Give a full proof of Theorem A.1. Lemma A.3 can be deduced from Lemma 4.3, while Lemma A.4 appears in [Chang 2002] as Lemma 3.1. You can follow Lovett's exposition: http://dx.doi.org/10.4086/toc.gs.2015.006

6. **[Jihyo Chae] A Sum-Product Estimate in Finite Fields, and Applications.**
   Jean Bourgain, Nets Katz and Terence Tao
   Link https://doi.org/10.1007/s00039-004-0451-1
   Remarks: Present the proof of Theorem 1.1.

7. **[Pauwel Van Den Eeckhaut] Sum-product in the reals.**
   Present the two papers
   László Székely, Crossing Numbers and Hard Erdős Problems in Discrete Geometry
   Link https://doi.org/10.1017/S0963548397002976
   György Elekes, On the Number of Sums and Products
   Link https://doi.org/10.4064/AA-81-4-365-367
   culminating in the proof of Theorem 1 in the second paper.

Tuesday (Inverse Gowers Theorems)

8. **[Lua Viana Reis] On large subsets of $\mathbb{F}_q^n$ with no three-term arithmetic progression**
   Jordan S. Ellenberg and Dion Gijswijt
   Journal link: https://annals.math.princeton.edu/2017/185-1/p08
   Remarks: Prove Corollary 5.

9. **[Aleksandar Bulj]Strong Bounds for 3-Progressions**
   Zander Kelley and Raghu Meka
   arXiv-Link: https://arxiv.org/abs/2302.05537
   Remarks: Follow the Bloom–Sisask exposition https://arxiv.org/pdf/2302.07211v3, focus on Theorems 2,4 in the finite field setting: Section 1 gives sketch of proof, present general results in Section 2, Section 3 proves Theorems 2,4. If time, discuss modifications needed in the integer setting.

10. **[Benjamin Gillot, Fredy Yip] A new proof of Szemerédi's theorem for arithmetic progressions of length four**
    W.T. Gowers
    Journal-link: https://link.springer.com/article/10.1007/s000390050065
    Remarks: Prove Theorem 20

11. **[Ferran Espuña] The Inverse Theorem for the U^3 Gowers Uniformity Norm on Arbitrary Finite Abelian Groups: Fourier-analytic and Ergodic**
    Asgar Jamneshan and Terence Tao
    arXiv-link: https://arxiv.org/pdf/2112.13759
    Remarks: Prove Theorem 1.6 by proving Lemmas 2.5, 2.6 and blackboxing Lemmas 2.2, 2.4 (but illustrate them using Example 2.3). The additive combinatorial input in the proof of Lemma 2.5 is covered in the Talks 3–5 and can be blackboxed. The finite field case of Lemma 2.5 is simpler (though following the general proof strategy) and covered in e.g. https://arxiv.org/abs/math/0604089.

12. **[Kate Thomas] An inverse theorem for the Gowers U^4 norm**
    Ben Green, Terence Tao and Tamar Ziegler
    Journal link: https://doi.org/10.1017/S0017089510000546
    Remarks: Present the arguments in Sections 6–9, with the goal of proving Theorem 1.5.

13. **[Daniel Zhu] Quasipolynomial inverse theorem for the $\mathsf{U}^4(\mathbb{F}_p^n)$ norm**
    Luka Milićević
    arXiv-link: https://arxiv.org/abs/2410.08966
    Remarks: Sketch the proof of Theorem 4. Treat Sections 4–7 in detail and then outline the main ideas of the remainder of the proof.

14. **[Oissin O'Donnell] Inverse and stability theorems for approximate representations of finite groups**
    W.T. Gowers and Omid Hatami
    arXiv-Link: https://arxiv.org/pdf/1510.04085
    Remarks: Sketch the proof of the scalar-valued U^2-inverse theorem on finite abelian groups highlighting the Fourier-theoretic identities needed therein. Compare this with the analogous properties in non-commutative Fourier analysis in Lemma 3.3 (you can omit their proofs). Prove Theorem 5.4 and the results needed in its proof in Sections 4 and 5.

Wednesday (HOFA and Sum Sets)

15. **[Jianghao Zhang] On a conjecture of Marton**
    W.T. Gowers, Ben Green, Freddie Manners, and Terence Tao
    arXiv-Link: https://arxiv.org/pdf/2311.05762
    Remarks: Prove Theorem 1.2 by proving Theorem 1.8 (include the deduction of Theorem 1.2 from Theorem 1.8 in Appendix B). To prove Theorem 1.8 the proof of the estimates in Sections 4.5 can be omitted.

16. **[Nihan Tanisali] An equivalence between inverse sumset theorems and inverse conjectures for the U^3 norm**
    Ben Green and Terence Tao
    arXiv-Link: https://arxiv.org/pdf/0906.3100
    Remarks: Prove Theorem 1.12 (one direction of the equivalence in Theorem 1.12 is sketched at the beginning of Appendix A, this relies on Talk 9) and the main result of Section 4. The equivalence in Theorem 1.12 is also established in this paper by Lovett https://arxiv.org/pdf/1001.3356 which you could use as an additional resource.

17. **[Hayk Aprikyan] A bilinear Bogolyubov-Ruzsa lemma with polylogarithmic bounds**
    Kaave Hosseini and Shachar Lovett
    Journal link: https://doi.org/10.19086/da.8867
    Remarks: Prove Theorem 1.3.

Thursday (Connections to CS)

18. [**Zimrat Kaniel**] **Higher-order Fourier analysis & algebraic property testing**
Hamed Hatami, Pooya Hatami and Shachar Lovett
Link: `https://cseweb.ucsd.edu/~slovett/files/survey-higher_order_fourier.pdf`
Remarks: Explain the BLR test and prove Theorem 2.3. Explain the relation between the Gowers uniformity norms and the AKKLR test (see Section 5.1). Sketch the proofs of Theorem 3.6 and Theorem 3.17. As an additional resource, you could use https://link.springer.com/article/10.1007/s11854-012-0018-2 (see Theorem 1.5).

19. [**Isaac Holt**] **Quadratic Fourier analysis and stabilizer testing**
Arunachalam–Dutt & Bao–van Dordrecht–Helsen
Proceedings-link 1: https://dl.acm.org/doi/pdf/10.1145/3717823.3718277
Proceedings-link 2: https://dl.acm.org/doi/10.1145/3717823.3718201
Remarks: Explain the connection between the U^3 norm and Theorem 1.1 of Arunachalam–Dutt based on the introduction up to (but not including) Section 1.3.1. Prove the U^3 inverse theorem for quantum states (ie., an L^2 version of the usual inverse theorem) based on Bao–van Dordrecht–Helsen. You may assume that the definition of the U^3 norm, the Balog–Szemerédi–Gowers theorem and the polynomial Freiman–Ruzsa theorem have been covered already.

20. [**Davi Castro Silva**] **Quadratic Goldreich–Levin theorems**
Jop Briët and Davi Castro-Silva
arXiv-Link: https://arxiv.org/abs/2505.13134
Remarks: Prove Theorem 1.1 and Corollary 1.3. You may assume that the uniformity norm, stabilizer states and the basics of symplectic geometry and the characteristic distribution have been covered.

21. [**Phillippe van Dordrecht**] **Stabilizer rank and higher-order Fourier analysis**
Farrokh Labib
Journal link: https://quantum-journal.org/papers/q-2022-02-09-645/pdf/
Remarks: Prove Theorem 1.1 and explain the link between higher-order Fourier analysis and stabilizer rank. You may assume that the definition of a stabilizer state has been covered. Note that this result was improved in https://dl.acm.org/doi/10.1145/3618260.3649733. Time permitting, you can sketch an outline of the proof.

22. [**Valeri Gladkova**] **The distribution of polynomials over finite fields, with applications to the Gowers norms**
Ben Green and Terence Tao
Journal link: https://doi.org/10.55016/ojs/cdm.v4i2.62086
Remarks: Prove Theorem 1.8.

23. [**Anthony Ostuni**] **Effective Bounds for Restricted 3-Arithmetic Progressions in $\mathbb{F}_p^n$**

Amey Bhangale, Subhash Khot and Dor Minzer
Journal link: https://arxiv.org/abs/2308.06600
Remarks: Prove Theorem 1.1. Treat the paper as self-contained; in particular, use Subsection 2.4 as a black box.

Friday (Applications to Algebraic Geometry and Number Theory)

24. [**Noa Bihlmaier, Nick Ruoff**] **Small subalgebras of polynomial rings and Stillman's Conjecture**
Tigran Ananyan and Melvin Hochster,
Journal link: https://doi.org/10.1090/jams/932
Remarks: Present the whole paper.

25. [**Tiago Moreira**] **The Möbius function is strongly orthogonal to nilsequences**
Ben Green and Terence Tao
Journal link: https://doi.org/10.4007/annals.2012.175.2.3
Remarks: Present the proof of Theorem 1.1. The nilsequences equidistribution theory, covered in the reference [9], should be used as a black box and only briefly outlined. On the other hand, as an illustration of the general theory, the special case of the Heisenberg example (treated in Section 5 of [9]) should be covered in detail.

26. [**Felix Pernegger**] **The Green-Tao theorem: an exposition**
David Conlon, Jacob Fox and Yufei Zhao
Journal link: https://doi.org/10.4171/emss/6
Remarks: Cover Sections 4–7 and prove Theorem 4.3. Explain how majorant in Section 8 is constructed and outline the calculation in Section 9.