DISCRETE DIRECTIONS IN HARMONIC ANALYSIS

ZANE KUN LI, DAMARIS SCHINDLER, RAJULA SRIVASTAVA, CHRISTOPH THIELE

1. Broad Directions

I. Recent developments in counting rational and lattice points near smooth manifolds using harmonic analysis. Connections to Diophantine approximation.
II. Estimates for Dirichlet polynomials and general exponential sums.
III. Consequences of Diophantine approximation and counting results to harmonic analysis.
IV. Restriction/Kakeya phenomena in discrete settings and Arithmetic Progression Problems
V. Discrete restriction

2. Papers

The instructions are either with respect to the latest arXiv version or the journal version of the paper, depending on whichever is provided.

I. Counting Rational Points near Smooth Manifolds.

(1) **Integral points close to a space curve**
Jing-Jing Huang
Journal Link: `https://doi.org/10.1007/s00208-019-01832-5`
Remarks: You can blackbox Lemmas 1-3, but it will be good to give a motivation for them (17 pages).

(2) **The density of rational points near hypersurfaces**
Jing-Jing Huang
Journal Link: `https://doi.org/10.1215/00127094-2020-0004`
Remarks: 33 pages, only Prove Theorem 1 (Section 5). If time, Diophantine applications (Section 4 and Section 9).

(3) **Lattice points on a curve via ℓ^2 decoupling**
Daishi Kiyohara
Journal Link: `https://doi.org/10.1142/S1793042124500994`
Remarks: Present the whole paper. You can state and use decoupling for the moment curve as a blackbox.

Date: Sep 2025.

II. Dirichlet series and Riemann zeta function.

(4) **On the Distribution of Dirichlet Sums**
Jean Bourgain
Journal Link: https://doi.org/10.1007/BF03341964
Remarks: The goal is to present Bourgain's observation that Montgomery's conjecture for Dirichlet series implies Kakeya. For further references: see the works mentioned in https://mathoverflow.net/q/137635 and Section 10 of Ben Green's notes: http://people.maths.ox.ac.uk/greenbj/papers/rkp.pdf

(5) **New large value estimates for Dirichlet polynomials: Part 1**
Larry Guth, James Maynard
arXiv Link: https://arxiv.org/abs/2405.20552
Remarks: Present Sections 1-7 (till Page 23). This covers the Introduction and the harmonic analysis component. Persons presenting parts of Guth-Maynard will need to work together to ensure a smooth transition between presentations.

(6) **New large value estimates for Dirichlet polynomials: Part 2**
Larry Guth, James Maynard
arXiv Link: https://arxiv.org/abs/2405.20552
Remarks: Present Sections 8-12 (Pages 24- 44). This is the additive combinatorics part and conclusion of the proof. If time permits, you can briefly comment on applications to the Riemann zeta function. Persons presenting parts of Guth-Maynard will need to work together to ensure a smooth transition between presentationsn.

III. Consequences of Diophantine approximation and counting results to harmonic analysis.

(7) **Bounds for the maximal function associated to periodic solutions of one-dimensional dispersive equations**
A. Moyua, L. Vega
Journal Link: https://doi.org/10.1112/blms/bdm096
Remarks: Present the entire paper (12 pages).

(8) **A note on maximal operators for the Schrödinger equation on $\mathbb{T}^1$**
Yuqiu Fu, Kevin Ren, and Haoyu Wang
arXiv Link: https://arxiv.org/abs/2307.12870
Remarks: Present the entire paper (13 pages).

(9) **An L^4 maximal estimate for quadratic Weyl sums**
Alex Barron
Journal Link: https://doi.org/10.1093/imrn/rnab182
Remarks: Present proof of Theorem 1.1 (Section 2 of paper)

(10) **L^p maximal estimates for quadratic Weyl sums**
Roger Baker
Journal Link: https://doi.org/10.1007/s10474-021-01173-3

Remarks: This paper (10 pages) reproves Barron's result using standard circle method machinery. Present the entire paper though feel free to cite standard lemma's from Vaughan's circle method book.

(11) **A note on the Schrödinger maximal function**
Jean Bourgain
Journal Link: https://doi.org/10.1007/s11854-016-0042-8
Remarks: See Pierce's exposition (*On Bourgain's counterexample for the Schrödinger maximal function*, 36 pages, https://doi.org/10.1093/qmath/haaa032) for additional understanding and explanation. Present Bourgain's paper (4 pages).

(12) **Uniform counterexamples to the convergence problem for periodic dispersive equations with a polynomial symbol**
Daniel Eceizabarrena, Xueying Yu
arXiv Link: https://arxiv.org/abs/2408.13935
Remarks: Present the entire paper (12 pages). Bourgain's counterexample was generalized here to equations of the form $iu_t + P(D)u = 0$.

(13) **Reversing a philosophy: from counting to square functions and decoupling**
P. Gressman, S. Guo, L. Pierce, J. Roos, and P-Y Yung
Journal Link: https://doi.org/10.1007/s12220-020-00593-x
Remarks: Only present the proof of Theorem 1.2 via Proposition 1.3 (leave out Section 3). Does Proposition 1.3 remind oneself of a particular lemma in number theory?

(14) **A distinction between the paraboloid and the sphere in weighted restriction**
Alex Iosevich, Ruixiang Zhang
arXiv Link: https://arxiv.org/abs/2312.12779
Remarks: Present the entire paper (17 pages).

IV. Restriction/Kakeya phenomena in discrete settings and Arithmetic Progression Problems.

(15) **Restriction and Kakeya phenomena for finite fields**
Gerd Mockenhaupt and Terence Tao
Journal Link: https://doi.org/10.1215/S0012-7094-04-12112-8
Remarks: Just present Page 35-52 (Sections 1-6, inclusive).

(16) **Finite field restriction estimates for the paraboloid in high even dimensions**
Alex Iosevich, Doowon Koh, and Mark Lewko
Journal Link: https://doi.org/10.1016/j.jfa.2019.108450
Remarks: Present the complete paper (16 pages)

(17) **On the size of Kakeya sets in finite fields**
Zeev Dvir
Journal Link: https://doi.org/10.1090/S0894-0347-08-00607-3

Remarks: Present the complete paper (5 pages).

(18) **The p-adic Kakeya conjecture**
Bodan Arsovski
Journal Link: `https://doi.org/10.1090/jams/1021`
Remarks: Present the complete paper (12 pages). Note that the journal version has quite a few differences from the arXiv version (namely the former also proves the maximal estimate), please read the journal version.

(19) **On large subsets of $\mathbb{F}_q^n$ with no three-term arithmetic progression**
Jordan S. Ellenberg, Dion Gijswijt
Journal Link: `http://dx.doi.org/10.4007/annals.2017.185.1.8`
Remarks: Present the entire paper (5 pages).

V. Discrete restriction.

(20) **Improved discrete restriction for the parabola**
Shaoming Guo, Zane Kun Li, Po-Lam Yung
Journal Link: `https://dx.doi.org/10.4310/MRL.2023.v30.n5.a4`
Remarks: One can assume the standard heuristics about the locally constant property/uncertainty principle (which are heuristics over $\mathbb{R}$ but rigorous over $\mathbb{Q}_p$ and start reading from Section 4. The key point will be to present the proof of Propositon 6.3 (Section 7 onwards).

(21) **Scale invariant Strichartz estimates on tori and applications**
Rowan Killip and Monica Visan
Journal Link: `https://dx.doi.org/10.4310/MRL.2016.v23.n2.a8`
Remarks: Just present Section 2.