Higher-order Fourier analysis and related topics

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1 List of Topics

Monday (Foundations)

1. **Roth's theorem**
   Text book: https://yufeizhao.com/gtacbook/
   Remarks: Study Chapter 6 up to Section 6.4. Prove Theorem 6.4.1.

2. **Szemerédi's regularity lemma and the triangle removal**
   Text book: https://yufeizhao.com/gtacbook/
   Remarks: Prove Theorem 2.1.9 and Theorem 2.3.1.

3. **New proofs of Plünnecke-type estimates for product sets in groups**
   Giorgis Petridis
   Journal link: https://link.springer.com/article/10.1007/s00493-012-2818-5
   Balog–Szemerédi–Gowers theorem
   Textbook: https://yufeizhao.com/gtacbook/
   Remarks: Present Sections 2 and 3 of the paper, culminating in the proof of Theorem 1.2. Concerning the book, study Section 7.13 and prove Theorem 7.13.6.

4. **Freiman's theorem**
   Text book: https://yufeizhao.com/gtacbook/
   Remarks: Prove Theorem 7.5.4 (Freiman's theorem in groups with bounded exponent). To this end, study Sections 7.1-7.5. Skip sum set things already covered in Talk 3.

5. **On the Bogolyubov—Ruzsa lemma**
   Tom Sanders
   Journal link: http://dx.doi.org/10.2140/apde.2012.5.627
   Remarks: Give a full proof of Theorem A.1. Lemma A.3 can be deduced from Lemma 4.3, while Lemma A.4 appears in [Chang 2002] as Lemma 3.1. You can follow Lovett's exposition: http://dx.doi.org/10.4086/toc.gs.2015.006

6. **On large subsets of $\mathbb{F}_q^n$ with no three-term arithmetic progression**
   Jordan S. Ellenberg and Dion Gijswijt

Journal link: https://annals.math.princeton.edu/2017/185-1/p08
Remarks: Prove Corollary 5.

Tuesday (Inverse Gowers Theorems)

7. **Strong Bounds for 3-Progressions**
   Zander Kelley and Raghu Meka
   arXiv-Link: https://arxiv.org/abs/2302.05537
   Remarks: Follow the Bloom–Sisask exposition https://arxiv.org/pdf/2302.07211v3, focus on Theorems 2,4 in the finite field setting: Section 1 gives sketch of proof, present general results in Section 2, Section 3 proves Theorems 2,4. If time, discuss modifications needed in the integer setting.

8. **A new proof of Szemerédi's theorem for arithmetic progressions of length four**
   W.T. Gowers
   Journal-link: https://link.springer.com/article/10.1007/s000390050065
   Remarks: Prove Theorem 20

9. **The Inverse Theorem for the U^3 Gowers Uniformity Norm on Arbitrary Finite Abelian Groups: Fourier-analytic and Ergodic**
   Asgar Jamneshan and Terence Tao
   arXiv-link: https://arxiv.org/pdf/2112.13759
   Remarks: Prove Theorem 1.6 by proving Lemmas 2.5, 2.6 and black-boxing Lemmas 2.2, 2.4 (but illustrate them using Example 2.3). The additive combinatorial input in the proof of Lemma 2.5 is covered in the Talks 3–5 and can be blackboxed. The finite field case of Lemma 2.5 is simpler (though following the general proof strategy) and covered in e.g. https://arxiv.org/abs/math/0604089.

10. **An inverse theorem for the Gowers U^4 norm**
    Ben Green, Terence Tao and Tamar Ziegler
    Journal link: https://doi.org/10.1017/S0017089510000546
    Remarks: Present the arguments in Sections 6–9, with the goal of proving Theorem 1.5.

11. **Quasipolynomial inverse theorem for the $\mathsf{U}^4(\mathbb{F}_p^n)$ norm**
    Luka Milićević
    arXiv-link: https://arxiv.org/abs/2410.08966
    Remarks: Sketch the proof of Theorem 4. Treat Sections 4–7 in detail and then outline the main ideas of the remainder of the proof.

12. **Inverse and stability theorems for approximate representations of finite groups**
    W.T. Gowers and Omid Hatami
    arXiv-Link: https://arxiv.org/pdf/1510.04085
    Remarks: Sketch the proof of the scalar-valued U^2-inverse theorem on finite abelian groups highlighting the Fourier-theoretic identities needed

therein. Compare this with the analogous properties in non-commutative Fourier analysis in Lemma 3.3 (you can omit their proofs). Prove Theorem 5.4 and the results needed in its proof in Sections 4 and 5.

Wednesday (HOFA and Sum Sets)

13. **On a conjecture of Marton**
W.T. Gowers, Ben Green, Freddie Manners, and Terence Tao
arXiv-Link: https://arxiv.org/pdf/2311.05762
Remarks: Prove Theorem 1.2 by proving Theorem 1.8 (include the deduction of Theorem 1.2 from Theorem 1.8 in Appendix B). To prove Theorem 1.8 the proof of the estimates in Sections 4.5 can be omitted.

14. **An equivalence between inverse sumset theorems and inverse conjectures for the U^3 norm**
Ben Green and Terence Tao
arXiv-Link: https://arxiv.org/pdf/0906.3100
Remarks: Prove Theorem 1.12 (one direction of the equivalence in Theorem 1.12 is sketched at the beginning of Appendix A, this relies on Talk 9) and the main result of Section 4. The equivalence in Theorem 1.12 is also established in this paper by Lovett https://arxiv.org/pdf/1001.3356 which you could use as an additional resource.

15. **A bilinear Bogolyubov-Ruzsa lemma with polylogarithmic bounds**
Kaave Hosseini and Shachar Lovett
Journal link: https://doi.org/10.19086/da.8867
Remarks: Prove Theorem 1.3.

Thursday (Connections to CS)

16. **Higher-order Fourier analysis & algebraic property testing**
Hamed Hatami, Pooya Hatami and Shachar Lovett
Link: `https://cseweb.ucsd.edu/~slovett/files/survey-higher_order_fourier.pdf`
Remarks: Explain the BLR test and prove Theorem 2.3. Explain the relation between the Gowers uniformity norms and the AKKLR test (see Section 5.1). Sketch the proofs of Theorem 3.6 and Theorem 3.17. As an additional resource, you could use https://link.springer.com/article/10.1007/s11854-012-0018-2 (see Theorem 1.5).

17. **Quadratic Fourier analysis and stabilizer testing**
Arunachalam–Dutt & Bao–van Dordrecht–Helsen
Proceedings-link 1: https://dl.acm.org/doi/pdf/10.1145/3717823.3718277
Proceedings-link 2: https://dl.acm.org/doi/10.1145/3717823.3718201
Remarks: Explain the connection between the U^3 norm and Theorem 1.1 of Arunachalam–Dutt based on the introduction up to (but not including) Section 1.3.1. Prove the U^3 inverse theorem for quantum states (ie., an L^2 version of the usual inverse theorem) based on Bao–van Dordrecht–Helsen.

You may assume that the definition of the U^3 norm, the Balog–Szemerédi–Gowers theorem and the polynomial Freiman–Ruzsa theorem have been covered already.

18. **Quadratic Goldreich–Levin theorems**
Jop Briët and Davi Castro-Silva
arXiv-Link: https://arxiv.org/abs/2505.13134
Remarks: Prove Theorem 1.1 and Corollary 1.3. You may assume that the uniformity norm, stabilizer states and the basics of symplectic geometry and the characteristic distribution have been covered.

19. **Stabilizer rank and higher-order Fourier analysis**
Farrokh Labib
Journal link: https://quantum-journal.org/papers/q-2022-02-09-645/pdf/
Remarks: Prove Theorem 1.1 and explain the link between higher-order Fourier analysis and stabilizer rank. You may assume that the definition of a stabilizer state has been covered. Note that this result was improved in https://dl.acm.org/doi/10.1145/3618260.3649733. Time permitting, you can sketch an outline of the proof.

20. **The distribution of polynomials over finite fields, with applications to the Gowers norms**
Ben Green and Terence Tao
Journal link: https://doi.org/10.55016/ojs/cdm.v4i2.62086
Remarks: Prove Theorem 1.8.

21. **Effective Bounds for Restricted 3-Arithmetic Progressions in $\mathbb{F}_p^n$**
Amey Bhangale, Subhash Khot and Dor Minzer
Journal link: https://arxiv.org/abs/2308.06600
Remarks: Prove Theorem 1.1. Treat the paper as self-contained; in particular, use Subsection 2.4 as a black box.

Friday (Applications to Algebraic Geometry and Number Theory)

22. **Small subalgebras of polynomial rings and Stillman's Conjecture**
Tigran Ananyan and Melvin Hochster,
Journal link: https://doi.org/10.1090/jams/932
Remarks: Present the whole paper.

23. **The Möbius function is strongly orthogonal to nilsequences**
Ben Green and Terence Tao
Journal link: https://doi.org/10.4007/annals.2012.175.2.3
Remarks: Present the proof of Theorem 1.1. The nilsequences equidistribution theory, covered in the reference [9], should be used as a black box and only briefly outlined. On the other hand, as an illustration of the general theory, the special case of the Heisenberg example (treated in Section 5 of [9]) should be covered in detail.

24. **The Green-Tao theorem: an exposition**
David Conlon, Jacob Fox and Yufei Zhao
Journal link: https://doi.org/10.4171/emss/6
Remarks: Cover Sections 4–7 and prove Theorem 4.3. Explain how majorant in Section 8 is constructed and outline the calculation in Section 9.