

The following is a list of the references I mentioned throughout the lecture along with some additional reference material which may help you get a broader view of the literature.

Lecture 1

1. Bottou, L. and Cun, Y., 2003. Large scale online learning. *Advances in neural information processing systems*, 16.
2. Marchenko, V.A. and Pastur, L.A., 1967. Distribution of eigenvalues for some sets of random matrices. *Mat. Sb.(NS)*, 72(114), p.4.
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5. ImageNET <https://image-net.org/about.php>
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7. Ben Arous, G., Gheissari, R. and Jagannath, A., 2021. Online stochastic gradient descent on non-convex losses from high-dimensional inference. *Journal of Machine Learning Research*, 22(106), pp.1-51.

Here are some references for single and multi-index models. This list includes some additional material to help you get a better grasp of the field but is by no means exhaustive.

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11. Tan, Y.S. and Vershynin, R., 2023. Online stochastic gradient descent with arbitrary initialization solves non-smooth, non-convex phase retrieval. *Journal of Machine Learning Research*, 24(58), pp.1-47.
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16. Lee, J.D., Oko, K., Suzuki, T. and Wu, D., 2024. Neural network learns low-dimensional polynomials with sgd near the information-theoretic limit. *Advances in Neural Information Processing Systems*, 37, pp.58716-58756.

Lecture 2:

Classical asymptotics

1. Robbins, H. and Monro, S., 1951. A stochastic approximation method. *The annals of mathematical statistics*, pp.400-407.
2. McLeish, D.L., 1976. Functional and random central limit theorems for the Robbins-Munro process. *Journal of Applied Probability*, 13(1), pp.148-154.
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4. Stroock, D.W. and Varadhan, S.S., 2007. *Multidimensional diffusion processes*. Springer.

Infinite width/Hydrodynamic scaling

5. Mei, S., Montanari, A. and Nguyen, P.M., 2018. A mean field view of the landscape of two-layer neural networks. *Proceedings of the National Academy of Sciences*, 115(33), pp.E7665-E7671.
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High-dimensional scaling

7. Saad, D. and Solla, S.A., 1995. Exact solution for on-line learning in multilayer neural networks. *Physical Review Letters*, 74(21), p.4337.
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Lecture 3

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