HAUSDORFF CENTER FOR MATHEMATICS — UNIVERSITÄT BONN

"Modern Methods in the Calculus of Variations"

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organized by
Alice Marveggio, Antonio Tribuzio, Konstantinos Zemas

Abstracts of mini-courses

Andrea Braides (University of Rome Tor Vergata)

Gamma-convergence and Non-Local Functionals

Abstract: One of De Giorgi's first observations on Gamma-convergence is that local integral functionals are a stable class under some very general polynomial growth conditions on the integrands. This is not the case for classes of nonlocal functionals, even for simple energies given by double integrals. We will analyze some problems, often set in fractional Sobolev spaces, in which the usual theories of homogenization, dimensional reduction, phase transitions, approximation of free discontinuity problems, discretization, etc., exhibit unexpected behaviors and pose new challenges.

Irene Fonseca (Carnegie Mellon University)

Phase Transitions of Heterogeneous Materials and Stochastic Homogenization in the $\mathcal{A}$-Free Setting

Abstract: In this series of lectures, using the notion of Γ-convergence as introduced by De Giorgi in 1975 we derive a variational model for phase transitions between two fluids as an asymptotic limit of a family of Cahn-Hilliard energies (also known as the Modica-Mortola functional, in the mathematical community). We will then consider a variational model for the interaction between homogenization and phase separation when small scale heterogeneities are present in the fluids. Three regimes will be considered: when the phase separation occurs at the same scale as the homogenization, when the latter is faster than the former (the supercritical case), and when homogenization is faster than the phase transition (the subcritial case).

As time will permit, we will address a compactness result for Γ-convergence of integral functionals defined on $\mathcal{A}$-free fields, leading to a theory of homogenization without periodicity assumptions that extends the "classical" context of energies depending on gradients, and ultimately this is used to study stochastic homogenization in the $\mathcal{A}$-free setting.

Carolin Kreisbeck (KU Eichstätt-Ingolstadt)

Nonlocal Gradients in Variational Problems - and How to Deal with Them

Abstract: Nonlocal variational models are used in various science and engineering applications, like continuum mechanics and image processing models. Yet, their analysis is far from straightforward, since classical methods in the calculus of variations, which tend to rely intrinsically on localization arguments, break down.

Motivated by new nonlocal models in hyperelasticity, this mini-course will explore a class of variational problems with integral functionals that depend on fractional and nonlocal gradients. After introducing these non-classical derivative operators, along with their basic properties and their associated Sobolev-type function spaces, we will discuss various aspects of the existence theory for such nonlocal problems and analyze their asymptotic behavior. In particular, we will address the tasks of identifying the appropriate notion of convexity for these problems, deriving new relaxation and homogenization results, and establishing localization limits, which provide a rigorous bridge between nonlocal and classical variational theories. An important ingredient in our analysis is the use of suitable translation operators, which enable us to pass between nonlocal and classical gradients and thus serve as effective technical tools for transferring results from one framework to the other.

Further topics to touch on include the characterization of functions with zero nonlocal gradients and nonlocal Neumann problems, heterogeneous nonlocal gradients with varying horizons and local boundary conditions, as well as rigorous linearization of models in nonlocal hyperelasticity.

These lectures are based mainly on a series of recent works with Hidde Schönberger (UC Louvain) and Javier Cueto (Universidad Autónoma de Madrid), and on ongoing joint work with Carlos Mora-Corral (Universidad Autónoma de Madrid), Felix Seifert (KU Eichstätt-Ingolstadt).

Xavier Lamy (University of Toulouse)

Around the Aviles-Giga Conjecture

Abstract: The Aviles-Giga energy is a phase transition model for gradient fields in two dimensions. It involves approximations of weak solutions to the eikonal equation $|\nabla u| = 1$. These approximations are assigned an energy cost depending on a transition scale $\epsilon \to 0$. At main order, the energy is conjectured to concentrate on the one-dimensional jump set of ∇u. A major difficulty is to understand the structure of the non-standard class of weak solutions (to the eikonal equation) selected by these approximations. In this minicourse, I will describe some of the ideas and methods generated by this open problem over the last three decades.

Abstracts of talks

André Guerra (University of Cambridge)

Differential Inclusions, the Monge-Ampère equation, and Morse Theory

Abstract: I will begin by giving a brief overview of rigidity and flexibility results in the theory of differential inclusions, a prime example being isometric embeddings. In two dimensions, the rigidity/flexibility of isometric embeddings is closely related to rigidity/flexibility of non-convex solutions to the Monge-Ampère equation. I will then discuss a recent result, obtained with R. Tione, which gives a complete rigidity/flexibility result for solutions of the Monge-Ampère equation in general dimension, as conjectured by Šverák in 1992. The proof relies on Morse theory for non-smooth functions.

Leonard Kreutz (Technical University of Munich)

Geometric Rigidity in Variable Domains and Applications in Dimension Reduction

Abstract: In this talk we present a quantitative geometric rigidity estimate in dimensions $d = 2, 3$ generalising a celebrated result by Friesecke, James and Müller to the setting of variable domains. Loosely speaking, we show that for each function $y \in H^1(\mathbf{U}; \mathbb{R}^3)$ and for each connected component of an open bounded set $\mathbf{U} \subset \mathbb{R}^d$, the L^2-distance of ∇y from a single rotation can be controlled up to a constant by its L^2-distance from the group $SO(d)$, with the constant not depending on the precise shape of $\mathbf{U}$, but only on an integral curvature functional related to $\partial\mathbf{U}$. We further show that for linear strains the estimate can be refined, leading to a uniform control independent of the $\mathbf{U}$. The estimate can be used to establish compactness in the space of generalized special funcions of bounded deformation (GSBD) for sequences of displacements related to deformations with uniformly bounded elastic energy. We show how this estimate can be applied in the context of dimension reduction by calculating the Γ-limits for thin elastic solids containing voids in different energy scaling regimes in terms of their thickness. This seminar is based on joint work with Manuel Friedrich (Johannes Kepler Universität Linz) and Konstantinos Zemas (Universität Bonn).

Theresa Simon (Universität Münster)

The Role of Linear Stability Analysis in Isoperimetric Problems

Abstract: While being a classical tool of the calculus of variations and ODE/PDE analysis, both in mathematical and physical contexts, linear stability analysis remains a crucial stepping stone towards the understanding of many nonlinear phenomena to this day. As an example, I will demonstrate its importance for (nonlocal) isoperimetric problems in a few instances: First, Hurwitz' classical proof of the isoperimetric inequality for curves in the plane boils down to it, and it can even be used to prove that any stable critical point must already be a disk. Turning towards developments of the last 20 years, I will then present how it lies at the heart of minimality of discs in nonlocal isoperimetric problems in almost local regimes, while the rigorous arguments proceed via quantitative isoperimetric inequalities. Finally, if time permits, I will briefly introduce tools for establishing linear stability in nontrivial settings.

Anna Skorobogatova (ETH Zürich)

Non-Uniqueness of Locally Minimizing Clusters via Singular Cones

Abstract: We study a variant of the multiple bubble problem in $\mathbb{R}^n$ with more than one infinite-volume chamber. Unlike the classical multiple bubble problem, this variant of the problem is global in nature, and crucially relies on understanding the geometry of the cluster at infinity. We provide an explicitly computable criterion that guarantees the existence of local minimizers with one finite-volume chamber and two infinite-volume chambers whose interface has a blowdown that is a singular minimizing cone. We verify that this criterion holds for a large number of dimensions $n \geq 8$, which in particular guarantees non-uniqueness of such local minimizers in such dimensions, in contrast to the case $n \leq 7$ which was settled by Bronsard and Novack. This is joint work with Lia Bronsard, Robin Neumayer and Mike Novack.
