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Tame fields and existential theories in equal characteristic

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Sixty years ago Ax, Kochen, and (independently) Ershov showed that the first-order theory of a field complete with respect to a non-Archimedean absolute value of characteristic 0 (e.g. $\mathbb{Q}((t))$ or $\mathbb{C}((t))$) is determined by the theories of its residue field and value group. The method is indirect: First, in order to facilitate the use of model theoretic methods, we allow valuations rather than absolute values - the key difference is that the image of the valuation is not required to be Archimedean. Second, we replace the hypothesis of completeness by considering instead those valued fields that are henselian, i.e. that satisfy the conclusion of Hensel's Lemma. So, for the class of henselian valued fields of equal characteristic zero, one proves a suitable embedding lemma: any pair of embeddings between residue fields and value groups extends to an embedding of valued fields, subject to a natural saturation hypothesis. This embedding lemma in fact goes through allowing constants from certain common subfields, and it yields right away an AKE principle at the level of existential theories. Thus, by a back-and-forth argument, the same holds for any other classical fragment: existential-universal sentences, etc. Perhaps the “state of the art”setting for such AKE principles is the theory of (separably) tame valued fields: these principles and the underpinning algebraic results are due in the main to Kuhlmann and his collaborators, principally Knaf and Pal. In this short course of five talks I hope to give a rather uniform presentation of AKE principles, beginning in the classical setting of equal characteristic zero, then extending to include those valued fields that are finitely ramified (in mixed characteristic) and (separably) tame. I will describe three families of extensions and applications. The first is to discuss what is known around AKE principles for certain expansions of the language of valued fields, notably difference fields and differential fields. The second is the analysis of existential theories of henselian valued fields, in equal characteristic and to a lesser extent in mixed characteristic. The third is that we lay the ground for the “Taming Theorem”of Jahnke and Kartas, where they find AKE results for the first time that apply to certain valued fields that admit finite extensions with nontrivial defect.

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