Book of Abstracts

Summerschool „Particles in Flow“

Armand Bernou

Control of correlation functions in mean-field systems and consequences

This talk will focus on mean-field particle systems with small, non-singular interactions and some noise. For such systems, ergodic properties of the mean-field limit PDE typically yield a uniform-in-time propagation of chaos property, which provides a first-order description of the system as N goes to infinity. I will discuss several approaches allowing to derive higher-order description of the system through a uniform-in-time control of many-body correlation functions. Some consequences on the phenomena of creation of chaos and Gibbs relaxation will also be mentioned. Based on joint works with Mitia Duerinckx and Matthieu Ménard.

Anne-Laure Dalibard

Modelization of congested flows

These lectures will be devoted to the modelization and mathematical analysis of flows in which the local density cannot take values above a certain threshold. Such models are used to describe phase transitions in fluids (coexistence of liquid and gas), crowd behavior, traffic flows... They are also strongly related to the description of floating structures.
We will first review different approaches to model congestion (at the microscopic or macroscopic level, with a hard or soft constraint...), and how these approaches can be related by asymptotic methods. We will then focus of macroscopic flows, and describe the interplay between the congested and non-congested regions.

Francisco Gancedo

One-phase contact point dynamics in Hele-Shaw cells

In this talk, we present a new result on the dynamics of 2D one-phase contact points in Hele-Shaw cells. Assuming vertical impermeable walls, we derive global-in-time energy estimates for initial data close to equilibrium. By exploiting the Neumann problem solved by the velocity potential, the analysis is performed in non-weighted L^2-based Sobolev spaces, without imposing restrictions on the size of the contact angles.

Megan Griffin-Pickering

Particle Approximations for the Vlasov-Poisson system for ions

The Vlasov-Poisson system for ions is a kinetic model for dilute plasma, describing electrostatic interactions between positive ions and thermalized electrons following a Maxwell-Boltzmann law. Compared to the well-known Vlasov-Poisson system for electrons, the ionic model features an additional exponential nonlinearity in the equation for the electrostatic potential, which creates several additional mathematical difficulties.

The equation arises formally as the mean field limit from an underlying microscopic system representing individual ions interacting with a thermalized electron distribution. However, when ions are modelled as point charges, it is an open problem to prove rigorously that the mean field limit holds.

I will discuss recent progress on the derivation of kinetic models for ionic plasma from 'regularized' microscopic systems, including a recent result in the style of the probabilistic approach developed by Lazarovici and Pickl for the electron Vlasov-Poisson system. The method developed allows interactions with nonlinear dependence on the particle density to be handled.

Matthieu Hillairet

Variational methods for the asymptotic analysis of Stokes system

In fluid/solid interactions, a crucial issue is the description of the flow of a fluid confined between two moving solid surfaces.
In this context, it is customary to consider the stationary Stokes problem of fluid mechanics supplemented with boundary conditions reproducing the displacement of the two surfaces.
The aim is then to obtain an asymptotic development of the solution when the distance between the two surfaces tends towards 0.

In this talk, I will present a variational method for discussing the asymptotics of the solution to Stokes'problem in the two-dimensional and three-dimensional cases.
We will see that these different cases present various difficulties inherent to the properties of divergence-free vector fields in each dimension.
The most recent result I will present have been obtained in collaboration with E. Bocchi, D. Bonheure, C. Patriarca and G. Sperone.

Richard Höfer

Homogenization of the Navier-Stokes equations in perforated domains

We consider the incompressible and compressible Navier-Stokes equations in domains perforated by particles on a grid with spacing $\eps$ and particle radii of order $\eps^\alpha$, \alpha \geq 1$. We study the limit of the solutions as $\eps \to 0$ depending also on scaling parameters of the Navier-Stokes equations, namely the Reynolds number, the Froude number and the Mach number. Depending on these parameters, we identify different regimes where the particles give rise to a macroscopic friction force, which leads to Darcy or Brinkman type equations. The results are obtained by relative energy methods.

Pierre-Emmanuel Jabin

A duality method for mean-field limits with singular interactions

Abstract: I will present a new approach to derive mean-field limits for first- and second-order particle systems with singular interactions. It is based on a duality approach combined with the analysis of linearized dual correlations, and it allows to cover for the first time arbitrary square-integrable interaction forces at possibly vanishing temperature. In case of first-order systems, it allows to recover in particular the mean-field limit to the 2d Euler and Navier-Stokes equations. The approach also provides convergence rates. This is based on a recent joint work with D. Bresch and M. Duerinckx.

Aline Lefebvre-Lepot

Close interactions in immersed granular flows

This presentation focuses on the simulation of dense granular media composed of macroscopic particles suspended in a viscous fluid, such as sludge. These systems exhibit highly complex macroscopic behaviors, including phenomena such as concentration instabilities and flow blockages.

At the heart of these behaviors are microscopic physical interactions between nearby particles—particularly lubrication effects caused by the interstitial fluid, as well as solid contact. These interactions are crucial not only for understanding immersed granular flows but are also highly relevant in the broader context of particle suspensions.

Despite their importance, the influence of these microscopic mechanisms on the overall dynamics of the system remains poorly understood. A key difficulty arises from the singular nature of these effects as the distance between particles approaches zero.

We begin by reviewing some results on the lubrication phenomenon between closely spaced particles, presenting asymptotic expansions that describes their behavior as inter-particle distances go to zero. We then show how these analytical results can be used to improve numerical simulations.

Then, because lubrication and contact effects introduce time singularities, we build on the asymptotic analysis to develop new contact models that take lubrication into account, allowing for the construction of stable and robust numerical schemes.

Finally, we show how these refined microscopic models can inform the design of one-dimensional macroscopic models, in which the non-overlapping constraint at the particle scale naturally leads to a maximum density condition in the continuum description.

Ayman Moussa

Weak-strong uniqueness for the 3D Vlasov-Navier-System

I will present some ongoing work with D. Han-Kwan, É. Miot, and I. Moyano, showing that all Leray solutions to the 3D VNS system coincide, as soon as a sufficiently strong solution exists for this system. Our proof combines the stability estimates we developed in [RMI 2020] for the 2D case with a clever trick by Chemin [CPAM 2011], which was previously used only for the NS system.

I will carefully avoid delving too much into Besov spaces during the talk, as — probably like you — I'm not particularly fond of them. However, I believe our main result can serve as a (modest) argument in favor of using these spaces, and I'll try to convince you of that!

References :

[CPAM 2011] : https://doi.org/10.1002/cpa.20386

[RMI 2020] : https://doi.org/10.4171/rmi/1120