

“Persistent Homology and Applications in Topological Data Analysis”

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organized by

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Abstracts

Kathryn Hess Bellwald (EPFL)

A Functor Calculus Perspective on Multi-persistence

Abstract: A multi-persistence module is functor from a poset (often taken to be a product of k totally ordered sets) to the category of vector spaces over a fixed field (often $\mathbb{F}_2$). As is well known, when $k > 1$, not all multi-persistence modules admit an interval decomposition, so there is no invariant analogous to the barcode of ordinary persistence modules. The analysis and characterization of multi-persistence modules thus requires a more sophisticated approach.

In this series of lectures, I will present the research program of my PhD student Bjørnar Hem, who has developed a framework for studying multi-persistence modules that is motivated by functor calculus. There are several well-known, widely used variants of functor calculus, each of which associates to a functor F of a particular type a "Taylor tower" of "degree n " functors $T_n F$, in a manner analogous to the Taylor series for functions, where the definition of "degree n " depends on the context. As n increases, the approximations $T_n F$ to F become better and better, in a sense relevant to the particular context. Under good conditions, the tower of approximations "converges" to F , and one can often deduce interesting properties of F from those of its Taylor approximations.

After a brief general overview of functor calculus, I will define poset calculus and cocalculus, provide examples of computations of approximations to multi-persistence modules, and discuss stability of approximations. I will then describe how poset calculus and cocalculus can be used to derive new decomposition results for multi-persistence modules. To conclude I will introduce a distinct but closely related (co)calculus for multi-persistence modules with a weaker notion of (co)degree n approximation, explore its relation to the poset (co)calculus, and highlight its close relation to projective dimension. Bjørnar will conclude the lecture series with work in progress and future perspectives for this research program.

Magnus Botnan (VU Amsterdam)

Inequalities for the Persistent Laplacian and Extremal Flag Filtrations

Abstract: Talk 1: Cheeger Inequalities for the persistent Laplacian

The persistent Laplacian was introduced independently by Lieutier et al. and Wang et al., and was later developed systematically by M\"{u}moli et al. as a spectral tool for studying persistent homology. In this talk, I will introduce the basic construction, with an emphasis on its relationship to classical combinatorial Laplacians and Dirichlet-to-Neumann operators. I will then discuss recent joint work with Rui Dong, in which we establish several Cheeger-type inequalities relating eigenvalues of the persistent Laplacian to geometric and combinatorial quantities associated with simplicial complexes.

Talk 2: Extremal Flag Filtrations

Given an edgewise filtration on n vertices. What is the maximal number of intervals in the Rips persistent homology barcode? What is the maximal total persistence? Which filtrations are extremal? In this talk, I will give complete answers to several of these questions and related extremal problems, and discuss a number of open directions. This is joint work with Lies Beers.

David Loiseau (Lawrence Berkeley National Laboratory)

Introduction to TDA for Machine Learning

Abstract: Modern machine learning often relies on finding useful representations of complex, structured data. For data with an underlying geometric structure such as point clouds, images, graphs, or time series, TDA provides tools for constructing representations that capture shape across multiple scales. In this minicourse, I will recall the intuitions of TDA, and introduce the associated statistical and machine learning framework. Then, I'll illustrate how topological features can be extracted from data and how the resulting descriptors can be integrated into typical machine learning pipelines. This includes unsupervised learning (vectorization), kernels (e.g., Sliced Wasserstein), and modern machine learning integration (e.g., autodiff handling). Finally, I'll generalize these constructions to the multiparameter persistence setting.

Yasuaki Hiraoka (Kyoto University)

Applications of Persistent Homology to Materials Science

Abstract: In this lecture, I will discuss some practical applications of persistent homology to materials science. In particular, I will focus on inverse analysis, machine learning, and representation theory in the context of persistent homology.

Contributed Talks

Julian Brüggemann (IMPAN)

Discrete Morse Theory in Topological Data Analysis

Abstract: Discrete Morse theory is a versatile tool from combinatorial algebraic topology. In a nutshell, discrete Morse theory uses certain well-behaved real-valued functions on face posets of regular CW complexes to study topological features. These discrete Morse functions provide sublevel filtrations, collapsing orders, and numerous tools to find theoretic guarantees and optimizations for various algorithms in TDA.

In this talk, we provide a short introduction to discrete Morse theory and consider a couple of applications, where discrete Morse theory has proved to be useful for topological data analysis.

Divya Ahuja (The Institute of Mathematical Sciences, Chennai)

Persistent Simple-Homotopy Invariants via Discrete Morse Theory and Whitehead Torsion

Abstract: Persistent homology captures the evolution of homological features along a filtration, but it does not record finer information related to simple-homotopy type. In this talk, I will discuss two approaches to studying such information in filtrations. First, I will introduce a Morse complexity profile based on the minimal number of critical simplices in discrete Morse functions and discuss computable approximations and their stability. I will then introduce persistent Whitehead torsion on filtration windows where the structure maps are homotopy equivalences, together with its behavior under interleavings.

Enhao Liu (Kyoto University)

Interval Invariants of Multiparameter Persistence: Multiplicities, Replacements, and Resolutions

Abstract: The interval decomposition of ordinary persistent homology motivates studying interval modules in multiparameter persistence. I will present results of computing interval multiplicities from projective resolutions or injective coresolutions of multiparameter persistent homology, together with criteria for filtering candidate intervals. This workflow is implemented in a Python project called MPIntInv.

To approximate general persistence modules using interval-decomposable modules, I will discuss algebraic approaches such as interval replacements and, if time permits, interval resolutions.

An Algebraic Method for Deriving Central Limit Theorems for Persistent Betti Numbers

Abstract: Various persistent homological invariants have been developed in recent years, raising the question of whether their probabilistic behavior can be understood in a unified way. In this talk, I will introduce a method based on homological algebra for deriving central limit theorems for persistent Betti numbers associated with $\mathbb{R}$ -indexed chain complexes constructed from a homogeneous Poisson point process. The key point is that the translation invariance, weak stabilization, and bounded moment assumptions in the Penrose–Yukich framework can be verified by studying the kernels and cokernels of add-one maps between chain complexes. As applications, we recover the known central limit theorem for persistent Betti numbers arising from simplicial complex filtrations and establish central limit theorems for blurred magnitude homology and magnitude Betti numbers of random geometric graphs.
